LATTICE STRUCTURES FOR 2-D NON-SEPARABLE OVERSAMPLED LAPPED TRANSFORMS

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ABSTRACT

- Novel lattice structures are proposed for constructing 2-D non-separable oversampled lapped transforms (NSOLTs).
- The structures can take RATIONAL REDUNDANCY as well as the overlapping, parunitary, linear-phase, real-valued and FIR property.
- The redundancy can be set to less than that of the nonsubsampled Haar wavelets while keeping the image restoration quality better.

Key words— Tight frame, LPPUFB, Directional transform, Lattice structure, Image restoration

INTRODUCTION

A parallel structure of a P-channel filter bank

- Non-separable (NS) property allows filters to be directional.
- Oversampled (OS) systems (R > 1) are superior to critically-sampled (CS) ones (R = 1) in terms of sparse representations.
- Image restoration is a good example of applications.
- Two popular ways to construct redundant systems are
  - Mixture construction of CS systems
  - Nonsubsampled construction of a CS system

Simple, but able to take only INTEGER REDUNDANCY

We propose a new class of NS OS systems, NSOLTs.

- Extension of OSLPPRFB to the NS case
- Generalization of NSLPPRFB to the OS case

2-D NON-SEPARABLE OVERSAMPLED LPPRFB

Summary of LPPR FIR filter banks, where 'NS', 'OS', 'LP' and 'PU' denote 'non-separable', 'oversampled', 'linear phase', and 'parunitary', respectively. 'P' denotes 'possible.'

![Image 1](https://via.placeholder.com/150)

<table>
<thead>
<tr>
<th>LPPR FIR FBs</th>
<th>NS</th>
<th>OS</th>
<th>LP</th>
<th>PU</th>
</tr>
</thead>
<tbody>
<tr>
<td>DCT/ Haar DWT</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>5-3 DWT / 9-7 DWT</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>LPPRFB (Tran et al., TIP 2000)</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>P</td>
</tr>
<tr>
<td>NSLPPRFB (Tran et al., SP-LIP 2001)</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>P</td>
</tr>
<tr>
<td>OSLPPRFB (Tran et al., TIP 2000)</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>P</td>
</tr>
<tr>
<td>Contourlet (Do et al., TIP 2005)</td>
<td>Yes</td>
<td>Yes</td>
<td>Restricted</td>
<td></td>
</tr>
<tr>
<td>NSOLT (Proposal)</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>P</td>
</tr>
</tbody>
</table>

Theorem (1)

For a P-channel NS OS LPPR filter bank with sampling ratio \( M = M_M \), suppose all of the analysis and synthesis filters are of the same size \( L_k \times L_k = (N_k + 1)M_k \times (N_k + 1)M_k \). The number of symmetric filters \( p_k \) and antisymmetric filters \( p_k = P - p_k \) must satisfy the following necessary conditions.

1. When \( M \) is even and \( k = (N_k, N_k) \) is arbitrary, \( M/2 \leq p_k \leq P - M/2 \) and \( M/2 \leq p_k \leq P - M/2 \).
2. When \( M \) is odd and both of \( N_k \) and \( N_k \) are even, \( (M + 1)/2 \leq p_k \leq P - (M - 1)/2 \) and \( M - 1/2 \leq p_k \leq P - (M + 1)/2 \).
3. When \( M \) is odd and either of \( N_k \) or \( N_k \) is odd, \( (M + 1)/2 \leq p_k \leq P - (M - 1)/2 \) and \( M - 1/2 \leq p_k \leq P - (M + 1)/2 \).

Polyphase matrix is represented by propagation matrices as

\[ E(z) = G_k(1)(z)G_k(1)(z) \cdots G_k(1)(z)G_k(1)(z) \cdots G_k(1)(z)G_k(1)(z)E_0(z), \]

and theses systems are categorized into the following two types:

1. Type-I: \( p_k = p_k, N_k \neq 1, d \in (y, x) \)
2. Type-II: \( p_k \neq p_k, N_k \neq 1, d \in (y, x) \)

LATTICE STRUCTURES FOR NSOLT

- Type-I lattice structure of a 2-D NSOLT

Example 1: Critically-sampled Haar transform

The CS Haar transform as a special case

- Sampling ratio: \( M = M = 2 \)
- Channels: \( P = p = 2 + 2 = 4 \) \( (R = P/M = 2) \)
- Polyphase order: \( N = N = 1 \)

This system is PU (1-tight frame) and has no DC-leakage.

Example 2: Nonsubsampled Haar transform

The nonsubsampled Haar transform as a special case

- Sampling ratio: \( M = M = 1 \)
- Channels: \( P = p = 2 + 2 = 4 \) \( (R = P/M = 4) \)
- Polyphase order: \( N = N = 1 \)

This system is PU (1-tight frame) and has no DC-leakage.

Example 3: Type-II NSOLT with Rational Redundancy

A design example of Type-II NSOLT with rational redundancy

- Sampling ratio: \( M = M = 2 \)
- Channels: \( P = p = 5 + 2 = 7 \) \( (R = P/M = 7/4) \)
- Polyphase order: \( N = N = 4 \)
- Parameter matrices \( W_0, U_0, U_0 \) are orthonormal.

Designed to be PU (1-tight frame) with no DC-leakage.

EXPERIMENTAL RESULTS

Inpainting results with different transforms, where the iterative shrinkage/thresholding algorithm (ISTA) is adopted.

![Image 2](https://via.placeholder.com/150)

- Control parameter of ISTA was set to the best value for each case.
- Basis termination technique was applied as a boundary operation.

CONCLUSIONS

- Proposed novel lattice structures for 2-D NS LPPRFB, which can realize real-valued symmetric 1-tight frames.
- Verified the significance with redundancy less than two through image inpainting with ISTA.

Acknowledgment

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